THE ELEVENTH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Eleventh Annual Meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Thursday and Friday, December 29–30, 1904. The attendance included the following forty-nine members of the Society:

Mr. Joseph Allen, Dr. Grace Andrews, Professor Maxime Bôcher, Professor E. W. Brown, Dr. A. B. Coble, Professor F. N. Cole, Miss E. B. Cowley, Dr. D. R. Curtiss, Dr. W. S. Dennett, Mr. E. P. R. Duval, Dr. William Findlay, Professor T. S. Fiske, Dr. A. S. Gale, Miss Alice B. Gould, Professor J. G. Hardy, Professor James Harkness, Professor H. E. Hawkes, Professor L. I. Hewes, Dr. E. V. Huntington, Mr. S. A. Joffe, Dr. Edward Kasner, Professor C. J. Keyser, Professor P. A. Lambert, Mr. L. L. Locke, Professor E. O. Lovett, Dr. Max Mason, Professor W. H. Metzler, Mr. C. L. E. Moore, Professor E. H. Moore, Professor Frank Morley, Professor G. D. Olds, Miss Amy Rayson, Professor C. G. Rockwood, Professor E. D. Roe, Dr. Arthur Schultze, Mr. C. H. Sisam, Professor Virgil Snyder, Professor W. B. Smith, Dr. E. L. Stabler, Dr. C. E. Stromquist, President T. H. Taliaferro, Professor J. H. Tanner, Professor H. D. Thompson, Professor H. W. Tyler, Professor J. H. Van Amringe, Professor Anna L. Van Benschoten, Professor E. B. Van Vleck, Dr. E. B. Wilson, Mr. J. E. Wright.

A morning and an afternoon session were held on Thursday; a morning session on Friday sufficed to complete the work of the meeting. The retiring President of the Society, Professor Thomas S. Fiske, occupied the chair, being relieved by Professor G. D. Olds and Vice-President elect Professor E. W. Brown. At the opening of the afternoon session on Thursday, Professor Fiske delivered his presidential address on "Mathematical progress in America."

The Council announced the election of the following persons to membership in the society: Mr. G. I. Gavett, Stanford University; Mr. M. E. Graber, Heidelberg University, Tiffin, Ohio; Mr. E. B. Lytle, University of Illinois; Professor R. E. Moritz, University of Washington; Dr. B. L. Newkirk, University of California. Fourteen applications for membership in the Society were received.

A committee was appointed to arrange for the summer meeting and to consider the question of holding a colloquium in connection with that meeting.

Reports were presented by the Treasurer, Librarian, and Auditing Committee. These reports have appeared in the Annual Register. The membership of the Society has increased during the past year from 455 to 473. The number of papers presented at the meetings was 118; the total attendance of members at all meetings was 280; 160 members attended at least one meeting during the year. The library now contains about 2000 volumes; a complete catalogue is included in the Annual Register. The Treasurer's report shows a balance of $3884.28 on hand December 27, 1904; of this balance $1723.07 is credited to the life-membership fund.

An informal dinner on Thursday evening, attended by about thirty-five of the members, added much to the pleasure of the meeting.

At the annual election, which closed on Friday morning, the following officers and other members of the Council were chosen:

President,	Professor W. F. Osgood.
Vice-Presidents,	Professor E. W. Brown,
	Professor James Pierpont.
Secretary,	Professor F. N. Cole.
Treasurer,	Dr. W. S. Dennett.
Librarian,	Professor D. E. Smith.

Committee of Publication,

Professor F. N. Cole,
Professor Alexander Ziwet,
Professor D. E. Smith.

Members of Council to serve until December, 1907,

Professor E. R. Hedrick,	Professor E. O. Lovett,
Professor T. F. Holgate,	Professor L. A. Wait.

The following papers were read at the annual meeting:

(1) Dr. Max Mason: "The doubly periodic solutions of Poisson's equation in the plane."

(2) Professor Virgil Snyder: "On the forms of sextic scrolls having no rectilinear directrix."

(3) Dr. A. B. Coble: "Some applications of a theorem in the theory of forms."

(4) Professor L. E. DICKSON: "The group of a tactical configuration."

(5) Professor T. S. FISKE: Presidential address, "Mathematical progress in America."

(6) M. MAURICE FRÉCHET: "Sur les opérations linéaires (deuxième note)."

(7) Professor F. MORLEY: "On an inversive relation between five points of a plane."

(8) Mr. J. E. WRIGHT: "Application of the theory of continuous groups to a certain differential equation."

(9) Dr. EDWARD KASNER: "Geometry of point correspondences: osculating homographies."

(10) Mr. C. H. SISAM: "On septic scrolls."

(11) Dr. E. V. HUNTINGTON: "Note on the definitions of groups, abelian groups, and fields."

(12) Dr. E. V. HUNTINGTON: "A set of postulates for ordinary complex algebra."

(13) Dr. BURKE SMITH: "On the deformation of surfaces of translation."

(14) Professor L. E. DICKSON: "A general theorem on algebraic numbers."

(15) Dr. A. B. COBLE: "The similar projective groups of a cubic space curve and a quadric surface."

(16) Professor E. H. MOORE: "On a definition of abstract groups."

M. Fréchet's paper was communicated to the Society by Professor E. H. Moore. In the absence of the authors, M. Fréchet's paper was read by Dr. E. B. Wilson, and the papers of Professor Dickson and Dr. Smith were read by title. Professor Dickson's first paper and the papers of Mr. Wright and Dr. Smith appeared in the January number of the BULLETIN. President Fiske's address is contained in the present number. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Dr. Mason proved that the necessary and sufficient condition that Poisson's equation in the plane

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

have a doubly periodic solution with the periods a, b, where f has these periods, is that the integral of f carried over the

period rectangle of sides a, b is zero. A "doubly periodic Green's function" G was formed from sigma functions, its law of reciprocity developed, and the double periodic solutions of the differential equation expressed by quadrature in terms of f and G.

2. In this paper Professor Snyder completes the enumeration of sextic scrolls. Among the new forms added may be mentioned those having tacnodal curves of order 2, 3, 4, or 5, and a new form of developable, the latter having a cuspidal and a nodal quintic curve which have a common triple point. The method of point correspondence is used throughout; it is finally shown that if any two non-singular plane curves lie on an algebraic scroll, they must be projectively equivalent.

3. Dr. Coble's paper gives some applications of the following general theorem in the theory of forms: Any function homogeneous in the coefficients of each of a series of q-ary forms can be expressed *linearly* in terms of the coefficients of the concomitants of these forms. Hence interpreting a binary form of nth order as a point in a space of n dimensions, we may write down in terms of its concomitants of mth degree the most general spread of mth order. This permits of discussing easily the relations of such a spread to the *norm curve* in n dimensions. For example when $n = 3$ we derive the theorem: The most general algebraic surface of mth order which contains a cubic space curve as an asymptotic curve has $3(m-2)$ double points on the curve.

An application is also made to the theory of syzygies. If the above surface of mth order written down in terms of the coëfficients of *all* the concomitants of mth degree contains more constants than the greatest possible number, the existence of one or more syzygies between the concomitants is clearly indicated. Thus we obtain an arithmetical criterion for the location of syzygies. The method may be extended to include the case of any number of q-ary forms.

6. Let U_f be a linear operation applied to the continuous function $f(x)$. It is first noted that the continuous functions $k_n(y)$ which occur in the formulas

$$U_f = \lim \int_a^b k_n(y) f(y) dy,$$

expressing Hadamard's theorem, can be chosen as polynomials. From this may be derived a new development of U_f. Another more interesting development is furnished by one of Borel's theorems. This development gives the value of U_f, nothing being known about f but its values for all rational values of x. It is proved that, U_f being a linear operation applied to a continuous function $f(x)$ defined between a and b, and c a number between a and b, two linear operations V_{f_1}, W_{f_2} [determined by the respective values f_1, f_2 of f in (a, c) and (c, b)] can be found so that $U_f = V_{f_1} + W_{f_2}$.

Some remarks are added on the importance of the field of functions in which a linear operation is defined.

7. If the reflexions of a point d in the sides of a triangle abc form a triangle perspective with abc, then the relation of a, b, c, d is a mutual one. Moreover the relation of $abcd$ and the point ∞ is a mutual one. The paper of Professor Morley is occupied with the form of this relation, with the curve (a bicircular quartic) on which the fifth point lies when four are given, and with the determination of the four points when the curve is given.

9. An equation of the form $y = f(x)$ may be interpreted on the one hand as a curve in a plane, and on the other hand as a correspondence between the points of a straight line. In the latter interpretation, the line is regarded as a two dimensional manifold, with the point pair as element. The geometry of correspondence is capable of as many developments as is the theory of curves; Dr. Kasner considers, in particular, the theory of contact. The first approximation to an arbitrary correspondence in the neighborhood of a point pair is given by a similarity transformation; the second approximation is given by an homography which may be said to osculate the given correspondence. When the variables are interpreted in the complex plane, this result is obtained: In connection with an arbitrary conformal transformation of the plane, there exists, for each point, a unique osculating circular transformation.

10. Mr. Sisam determined and classified the septic scrolls having a rectilinear directrix. A one-to-one correspondence is established by pole and polar theory between the generators of the scroll and the points of a twisted curve. The types of scrolls are then determined from the types of curves thus ob-

tained. The classification was obtained geometrically, but the method of obtaining the equations of the scrolls was also given.

11. Dr. Huntington's first paper gives a summary of all the known sets of independent postulates for abstract groups, and adds the following set of six postulates, which is in some respects simpler than any of the previous sets:

1. If a and b are elements of the class, then their "product," ab, is an element of the class.

2. The associative law holds: $(ab)c = a(bc)$.

3. There is at least one element i such that $ii = i$; and

4. There is not more than one such element. (This unique element i is called the "identity" of the group.)

5. If i is the identity (see 3 and 4), then either $ia = a$ for every element a or else $ai = a$ for every element a. (Either alternative can be deduced from the other, in view of 1–6, so that it is not necessary to demand both.)

6. If i is the identity (see 3 and 4), then for every element a there is either an element a'_r such that $aa'_r = i$, or else an element a'_l such that $a'_l a = i$. (Either alternative can be deduced from the other, in view of 1–6, so that it is not necessary to demand both. The elements a'_r and a'_l are shown to be uniquely determined by a, and to be equal to each other; this element $a' = a'_r = a'_l$, is called the "inverse" of a.)

These six postulates are sufficient for the theory of abstract groups in general; if we wish to make the groups abelian, we must add a seventh postulate, namely:

7. The commutative law: $ab = ba$.

All seven postulates are independent, and remain independent if we add the demand that the group shall be infinite. If, however, we demand that the group shall be finite, postulates 3 and 6 become redundant. These seven postulates for an abelian group lead at once to a set of fifteen postulates for a *field*; a condensed list of ten postulates is also given. The paper will appear in the *Transactions*.

12. Dr. Huntington's second paper shows that a class K, with a relation $<$ and two operations $+$ and $\times$, will form a system abstractly identical with the algebra of ordinary complex quantities, when the following conditions are satisfied:

I. The class K is a field with respect to $+$ and $\times$.

II. The class K contains a subclass C which is also a field

with respect to $+$ and $\times$. (C corresponds to the class of reals.)

III. The subclass C is a one-dimensional continuum with respect to $<$ (see author's paper in the *Transactions* for January, 1905, § 1).

IV. Within the subclass C the following relations hold: 1) if $x < y$, then $a + x < a + y$; 2) if $a > 0$ and $b > 0$, then $a \times b > 0$ (where 0 is the zero element of the field).

V. There is an element j in K such that $j \times j = -1$ (where -1 is the negative of the unit element of the field); and if i is one of the elements which have this property, then for every element a in K there are elements x and y in C such that $x + iy = a$.

The postulates of these five groups, which number twenty-eight in all, are shown to be independent.

14. In his second paper, Professor Dickson considers the γ_{iks} of a multiplication table of a hypercomplex number system for the special case of algebraic numbers. Arranging the n^3 elements γ_{iks} in the form of a cube and taking the intersection by any plane parallel to a face, we obtain n^2 elements of non-vanishing determinant. This theorem that no one of the $3n$ determinants is zero is true for finite and algebraic fields, but not in general for other complex number systems. The problem arose in connection with an application to group theory.

15. By a method suggested by Professor Study, the integral equations of the projective three-parameter groups of a cubic space curve and of a set of generators of a quadric surface are obtained by Dr. Coble in such a form that the algebraic transformation T connecting them is evident. The effect of T upon the various manifolds is discussed and the results are used to obtain some of the properties of the algebraic group reciprocal to the projective group of the cubic curve.

16. Professor Moore improves the second of his two definitions of abstract groups given in the *Transactions* of October, 1902, by proving that the postulate $(3'_r)$ is redundant and that the remaining postulates $(1, 2, 3,'' 3''_r, 4''_l)$ are mutually independent.

F. N. COLE,
Secretary.

MATHEMATICAL PROGRESS IN AMERICA.

PRESIDENTIAL ADDRESS DELIVERED BEFORE THE AMERICAN MATHEMATICAL SOCIETY AT ITS ELEVENTH ANNUAL MEETING DECEMBER 29, 1904.

BY PRESIDENT THOMAS S. FISKE.

In the remarks that follow, I shall limit myself to a brief consideration of progress in pure mathematics. This I may do the more appropriately, inasmuch as one of my predecessors, Professor R. S. Woodward, at the annual meeting of 1899, gave an account of the advances made in applied mathematics during the nineteenth century. In his address, which was published in the BULLETIN for January, 1900,* is included a description of the more important advances made by Americans in the field of applied mathematics.

In tracing the development of pure mathematics in America, it seems convenient to recognize three periods. The first period extends from colonial days up to the establishment of the Johns Hopkins University in 1876; the second period extends from the establishment of the Johns Hopkins University up to 1891, when the New York Mathematical Society took on a national character and began the publication of its BULLETIN; the third period extends from 1891 up to the present time.

The most valuable source from which the general reader may secure information in regard to the first period, is a work entitled The Teaching and History of Mathematics in the United States.† This work, written by Professor Florian Cajori, was published in 1890 by the United States Bureau of Education.

Before the founding of Johns Hopkins University there was almost no attempt made to prosecute or even to stimulate in a systematic manner research in the field of pure mathematics. Such mathematical journals as were published were scientifically of little importance and as a rule lived but a year or two. The only exception that we need mention was the *Analyst*, edited by Dr. J. E. Hendricks and published at Des Moines, Ia., from 1874 to 1883; and the publication of this journal began practically at the close of the period referred to above.

* BULLETIN, series 2, vol. 6, pp. 133–163.

† U. S. Bureau of Education, Circular of Information No. 3, 1890.

However, there were a certain number of men, for the most part self-trained, who were eminent among their fellows for their mathematical scholarship, their influence upon the younger men with whom they came in contact, and their capacity for research. Of these the most conspicuous were Adrain, Bowditch, and Peirce. Adrain is known for his apparently independent discovery of the law of distribution of errors; Bowditch is known for his translation of Laplace's Mécanique Céleste, accompanied by a commentary of his own; and Peirce is now known chiefly for his classical memoir, Linear Associative Algebra, which was the first important research made by an American in the field of pure mathematics.

With the arrival of Professor Sylvester at Baltimore, and the establishment of the *American Journal of Mathematics*, began the systematic encouragement of mathematical research in America. Professor Sylvester drew about him a body of deeply interested students, and through his own untiring efforts and his inspiring personality a most powerful stimulus was exerted upon the mathematical activities of all who were associated with him. His work in this country, however, continued only seven years. In 1884 he returned to England to take the chair offered to him by Oxford University.

The first ten volumes of the *American Journal of Mathematics*, published from 1878 to 1888, contained papers contributed by about ninety different writers. Of these thirty were mathematicians of foreign countries. Almost one-third of the remaining sixty were pupils of Professor Sylvester; the others were mathematicians some of whom had come under the influence of Benjamin Peirce, some of whom had been students at German universities, and some of whom were in large degree self-trained. They seemed to need only an opportunity of publication and a circle of readers to induce them to rush into print. In fact several of them had already sent papers abroad for publication in foreign journals. Among the contributors to early volumes of the *American Journal of Mathematics* we should especially mention Newcomb, Hill, Gibbs, C. S. Peirce, McClintock, Johnson, Story, Stringham, Craig and Franklin.

We must at this point make some mention of the rapidly increasing influence of the German universities upon American mathematical activity. For some time a considerable number of young Americans, attracted by the superior opportunities offered by the German universities, had been going abroad for

the study of the more advanced branches of mathematics. The lectures of Professor Klein were in particular the Mecca sought by young Americans in search of mathematical knowledge. I think that it may be said safely that at present ten per cent of the members of the AMERICAN MATHEMATICAL SOCIETY have received the doctorate from German universities, and that twenty per cent of its members have for some time at least pursued mathematical studies in Germany. It is not surprising that as a result a large portion of the American mathematical output shows evidence of direct German influence, if not of direct German inspiration.

In 1883, as we have already indicated, the publication of the *Analyst* was discontinued. In the following year a new journal, the *Annals of Mathematics*, under the editorial management of Professor Stone of the University of Virginia, began publication. This journal was of a somewhat less ambitious character than the *American Journal of Mathematics*. It is interesting to note in connection with it that to a considerable extent its pages were given to papers on applied mathematics. In 1899 the *Annals* passed into the editorial control of the Mathematical Department of Harvard University. Since that time it has been largely expository or didactic. It has not sought to publish new investigations of an extended character, although it has not hesitated to publish brief papers announcing new results.

Let us now turn to a brief outline of the history of the Society which brings us together on this occasion.

At a meeting held November 24, 1888, six members of the Department of Mathematics of Columbia University formed a society which was to meet monthly for the purpose of discussing mathematical topics and reading papers of mathematical interest. At the meeting held a month later they resolved to call their society the New York Mathematical Society and to invite the coöperation of all persons living in or near New York City who might be professionally interested in mathematics. By the end of the year 1889 the membership of the Society had increased to sixteen. By the end of 1890 it had increased to twenty-two.

At the meeting held in December, 1890, the first president, Professor J. H. Van Amringe, retired from office, and Dr. Emory McClintock was elected his successor. At the same meeting the publication of a mathematical bulletin was pro-

posed. The officers of the Society a month later made a report in which they recommended that this bulletin, if established, should not seek to enter into competition with the existing mathematical journals, but that it should be devoted primarily to historical and critical articles, accounts of advances in different branches of mathematics, reviews of important new publications, and general mathematical news and intelligence. They showed at the same time that the expense connected with such a publication would necessitate an extension of the membership of the Society together with an increase in the annual dues. It was suggested, accordingly, that a general circular be issued, describing the aims of the Society and inviting suitable persons to become members.

After hearing the report, the Society authorized the secretary to undertake a preliminary correspondence with a few of the principal mathematicians of the country with a view to determining whether their favor and assistance might be secured for the proposed enterprise. A month later the secretary reported that he had received favorable responses from Professor Simon Newcomb, Professor W. Woolsey Johnson, Professor Thomas Craig and Professor H. B. Fine. As a result of these favorable responses, approval was given to the plan recommended by the officers of the Society for the extension of its membership and for the publication of a historical and critical review of pure and applied mathematics. A circular letter of invitation such as had been recommended was issued shortly thereafter. The proposals which it contained seemed to meet with general favor, and by June, 1891, the membership of the Society had risen to one hundred and seventy-four. The first number of the BULLETIN was issued in October, 1891. Its appearance increased the interest already excited, and by the summer of 1892 the membership of the Society had risen to two hundred and twenty-seven.

Professor Klein and Professor Study, who visited the United States in 1893 for the purpose of attending the International mathematical congress held in Chicago, were present at the meeting of the Society held in October of that year. They both delivered addresses before the Society and expressed great interest in its work.

By the spring of 1894 it was felt generally that the operations of the Society had assumed a national character, and a new constitution was adopted providing for a change of name

from the New York Mathematical Society to the AMERICAN MATHEMATICAL SOCIETY. In June of the same year the Society undertook to provide means for the publication of the papers read at the Chicago Congress the preceding year, and arrangements were made for holding in conjunction with the Brooklyn meeting of the American association for the advancement of science the first "summer meeting" of the Society.

At the annual meeting held December, 1894, Dr. Emory McClintock retired from the presidency, being succeeded by Dr. George W. Hill. At this meeting Dr. McClintock delivered an address which was published in the BULLETIN for January, 1895.* It was entitled "The past and future of the Society" and contains an account of the Society during the first six years of its existence. Upon the occasion of Dr. McClintock's retirement from the presidency the Society adopted a resolution expressing its appreciation of the great services that he had rendered while presiding officer, and its recognition of the fact that largely to his initiative were due the broadening of organization and extension of membership which made the Society properly representative of the mathematical interests of America.

The next event of special importance in the history of the Society occurred in 1896. Immediately after the summer meeting of that year, which was held in connection with the Buffalo meeting of the American association for the advancement of science, the Society's first "colloquium" took place. Interesting and instructive courses of lectures were delivered by Professors Bôcher and Pierpont, and at the close of the colloquium those participating in it recommended that similar arrangements be made periodically in connection with subsequent summer meetings. In the same year for the regular October meeting was substituted a special meeting at Princeton in connection with the sesquicentennial celebration of Princeton University. At that meeting addresses were delivered by Professor Klein and Professor J. J. Thomson.

In the spring of 1897 the Chicago Section of the Society was established. At the same time it was determined to replace the meetings held monthly in New York by meetings held four times a year at intervals of two months. The summer meeting of 1897 was held at Toronto in connection with the meeting of the British association for the advancement of science. This meeting was attended by a number of distinguished

* BULLETIN, series 2, vol. 1, pp. 85–94.

visitors from Great Britain, among whom were Professors Forsyth, Greenhill, and Henrici.

A colloquium was held in the summer of 1898 at Harvard University. There was much discussion among those attending it in regard to the need of larger and better facilities for the publication of mathematical researches. The following winter the Society proposed to the Johns Hopkins University that the *American Journal of Mathematics* should be enlarged and issued more frequently and that the Society should be given a share in the editorial control of the *Journal*. It was found impossible, however, to reach an agreement with the Johns Hopkins University, and in April, 1899, the Society determined to establish an organ of its own for the publication of the more important original papers presented at its meetings. The financial resources of the Society were not sufficient to carry on the work already begun and at the same time to provide for the new publication; but it was found possible to secure assistance from ten colleges and universities which promised to join in support of the undertaking. The new publication, known as the *Transactions of the American Mathematical Society*, made its first appearance in January, 1900.

Simultaneously with the meeting held in October, 1899, was held the first meeting of the newly organized American Physical Society. On this occasion the Mathematical Society met with the Physical Society for the purpose of listening to the address of President H. A. Rowland, of the Physical Society. Again, two months later, on the occasion of the annual meeting of the American Mathematical Society, the two societies met in joint session for the purpose of listening to the presidential address of Professor R. S. Woodward, of the Mathematical Society. In this connection it may be of interest to reeall that the organization of the American Physical Society was modelled, in a general way, after that of the Mathematical Society. The two societies, which have many members in common, have enjoyed uninterruptedly the most cordial relations.

In 1901 the Mathematical Society was compelled to turn its attention to the management of its rapidly growing library. An agreement was made with Columbia University whereby the library was entrusted to the care of that institution. The University undertook to bind and catalogue the books belonging to the Society and to make the arrangements necessary for the loan of the books to members. In return therefor the Uni-

versity is permitted to make use of the Society's collection in the way of a reference library. In October, 1901, the American Physical Society met again in joint session with the Mathematical Society for the purpose of listening to a paper by Professor Hadamard, who was visiting America as a delegate to Yale's bicentennial celebration.

In May, 1902, the San Francisco Section of the Society was established. In December of the same year Professor E. H. Moore delivered his address* as president of the Society. Very largely as a result of this address, the influence of the Society was exerted to bring about the organization of associations of teachers of mathematics with a view to improving the methods of mathematical teaching. The Association of teachers of mathematics in New England was organized in April, 1903. The Association of teachers of mathematics in the Middle States and Maryland was organized in November of the same year. Several similar associations have been organized more recently in various sections of the country.

Two events have occurred during the year now closing which are of sufficient importance to deserve mention here. One is the determination of the Society to publish in book form the mathematical lectures delivered at the colloquium held in Boston in 1903. The other is the meeting held last summer in connection with the International scientific congress at St. Louis.

In connection with this brief survey of the Society's history it seems desirable to indicate in figures the growth of the Society and its work. During the past ten years the membership of the Society has doubled, rising from about 250 in 1894 to almost 500 at the present time. Ten years ago the number of papers presented each year at the meetings of the Society was in the neighborhood of 30, of which about a dozen were subsequently published. During the year ending July 1, 1904, the number of papers presented at meetings of the Society was 154, and the number of papers published by its members was 85. In January 1902, when the present administration of the Society's library began, the number of volumes in the library was 121, while at present it is almost 2000.

If any one wishes to have pass before him in review the scientific activities of the Mathematical Society, he has only to consult two pamphlets issued a few months ago. I refer to the general index to the first thirteen volumes of the Society's

* BULLETIN, series 2, vol. 9 (1903), pp. 402-424.

BULLETIN, compiled by Dr. Emilie N. Martin, and to the index to the first five volumes of the *Transactions* compiled under the direction of the editor-in-chief of the *Transactions*. The Society today serves to bring together into a harmonious whole all the mathematical activities of America. It is only infrequently that a mathematical paper of importance is published without having been read previously at one of its meetings. To give an account of the present condition of the Society is practically the same as to give an account of the present condition of American mathematics.

Notwithstanding the great progress recently made in America by our science, we are far from being in a position that we can regard as entirely satisfactory. We have only to look about us in order to see that improvement is not only possible but necessary in almost every direction.

In the first place, the most pressing demand seems to be that those engaged in lecturing on the more advanced branches of mathematics at American universities should be given greater opportunities for private study and research. At present, the time of almost every university professor is taken up to a very large extent with administrative matters connected with the care of comparatively young students. Discussions in regard to admission requirements, the course of study, discipline, and the control of athletics, absorb a large part of the time and strength of the faculty of every university. It is possible that this situation will in the course of the next twenty years be greatly relieved by a change, which many consider is already in sight. This change is nothing more nor less than the relegation of the first two years of the ordinary college course to the secondary schools and the establishment of university courses that will begin with the present third year of the college. The progress made in recent years by the public high schools makes it plain that before long they will be able without difficulty to duplicate the first two years of the present college course, and as more highly trained teachers enter these schools there is no doubt that there will be a constantly increasing effort to take up college work. If this be done, not only will the condition of the secondary schools be greatly improved, but our university teachers will secure the relief so greatly needed for the advancement of the highest interests of our science.

In the second place, it is of the greatest importance that the mathematical journals already established in this country — the BULLETIN, the *Annals of Mathematics*, the *American Journal of Mathematics*, and the *Transactions* should all be encouraged and assisted to extend their influence and increase their efficiency. It is the duty of every member of the Society to interest himself to the greatest possible extent in the work of each of these journals. It is important also that we should strive to secure for these journals more adequate financial support. In other countries it is not unusual for the government itself to give financial support to such publications.

In the third place, we must have improved methods of teaching, better textbooks, and more good treatises on advanced subjects. The members of the Society, working as individuals, can do much along these lines. The Society as a whole, let us hope, will some day be able to render important assistance in the publication of mathematical works of the best type. It is quite possible that in some cases direct translation from foreign languages would be highly beneficial. Many of the most important mathematical works published in German, in French, or in Italian are at once translated so as to be accessible in all three of these languages. Is there no lesson in this for us? An English translation of the new German encyclopedia of mathematics would probably do much to spread throughout this land of seventy-five million inhabitants a knowledge of, and an interest in, advanced mathematics.

Finally, we must not relax our efforts to increase and improve the opportunities offered those interested in mathematics to meet one another for the purpose of exchanging their views upon mathematical topics. The Society must encourage, even to a greater extent than hitherto, the holding of mathematical colloquiums, sectional meetings, largely attended general meetings, and international congresses.

NEW YORK,
December, 1904.

THE HEIDELBERG CONGRESS: SECTIONAL MEETINGS.

(*Continued from page* 217.)

Section IV. *Applied Mathematics.*

Presidents: Klein, Runge, and Hauck. Chairmen: Van Vleck, Volterra, Hadamard, Börsch, and Finsterwalder.

(1) KLEIN (Göttingen): "Ueber die Probleme der angewandten Mathematik, besonders über die pädagogische Seite." †

(2) DELAUNAY (Warsaw): "Ueber das Problem der drei Körper." †

(3) LEVI-CIVITA (Padua): "Ueber das Problem der drei Körper." †

(4) WEINGARTEN (Freiburg): "Ueber einen Fall der Bewegung einer schweren Flüssigkeit mit freier Oberfläche."

(5) VOLTERRA (Rome): "Théorie des ondes."

(6) HADAMARD (Paris): "Sur les équations aux dérivées partielles de la physique mathématique." †

(7) SOMMERFELD (Aachen): "Mechanik der Elektronen." †

(8) GENESE (Aberyswith): "On the development of the Ausdehnungslehre by the principles of statics." †

(9) WEBER (Strassburg): "Einige Bemerkungen zum Vortrag Hadamards." †

(10) ANDRADE (Besançon): "Chronometrische Untersuchungen."

(11) BÖRSCH (Potsdam): "Heutige Kenntnis der Erdgestalt."

(12) FINSTERWALDER (Munich): "Flüchtige Aufnahmen mittels Photogrammetrie." †

(13) PRANDTL (Hanover): "Flüssigkeitsbewegung mit kleiner Reibung." †

(14) KEMPE (Rotterdam): "Ein Gelenkmechanismus." †

(15) RUNGE (Hanover): "Leibniz' Rechenmaschine." †

(16) DISTELI (Strassburg), SCHILLING (Danzig †), and WIENER (Darmstadt): "Communications and demonstrations on models collected in connection with the Congress."

1. In his opening address Klein said that if we compare mathematics to a fortress, the different branches of applied mathematics are represented by the outworks surrounding the

fortress on all sides and connecting it with the outer world. Common to all branches of applied mathematics is then only this, that the mathematical idea in them connects itself necessarily and inseparably with other lines of human interest. Applied mathematics stands in express contrast to those branches of our science which may be considered the citadel of the fortress, to formal mathematics (in the sense of Leibniz), that is to that treatment of mathematical questions which so far as possible ignores any concrete significance of the quantities and symbols employed and concerns itself only with the laws according to which the quantities and symbols may be combined. The welfare of the science requires unquestionably the free development of all its parts. Applied mathematics here undertakes the double rôle of ever conducting to the center from without the stimulus of new problems and conversely of securing for the product of the center due external efficiency. The significance of mathematics within the wide domain of human interest depends therefore on the active vitality of applied mathematics. Hence there where most abundant opportunity is given for the influence of mathematics on wider circles — that is, in the instruction of the youth — we must bring the teaching of applied mathematics to special prominence. From this point of view may be mentioned a recent paper (presented to section VI), "Ueber eine zeitgemässe Umgestaltung des mathematischen Unterrichts an den höheren Schulen." The object of this paper is to exhibit the theory of functions in such manner and with such illustration chosen from mechanics and physics as will warrant its introduction (as the centralizing agent for all of the more difficult arithmetical questions) into the curriculum of the higher schools. The paper is but a part of a collection soon to be printed on this general subject by a number of well known scientists.

2. When three masses m_1, m_2, m_3 move under the Newtonian law of attraction, the resultant forces pass through the same point which Delaunay calls the center of attraction. This is situated at the center of gravity of fictive masses $m_1\alpha^3$, $m_2\beta^3$, $m_3\gamma^3$ placed at the points m_1, m_2, m_3 (α, β, γ are the sides of the triangle m_1, m_2, m_3). If the coordinates of the masses are x_i, y_i, z_i ($i = 1, 2, 3$), those of the center of attraction may be expressed by

$$x = \Sigma ma^3 x/\Sigma ma^3 = \xi/\omega, \quad y = \Sigma ma^3 y/\Sigma ma^3 = \eta/\omega,$$
$$z = \Sigma ma^3 z/\Sigma ma^3 = \zeta/\omega.$$

The differential equations take the form

$$(x - x_1)\omega = \beta^3\gamma^3 \frac{d^2x_1}{dt^2}, \quad (y - y_1)\omega = \beta^3\gamma^3 \frac{d^2y_1}{dt^2},$$
$$(z - z_1)\omega = \beta^3\gamma^3 \frac{d^2z_1}{dt^2},$$

and similar equations obtained by permuting 1, 2, 3 and a, β, γ cyclically. These equations can be utilized in seeking the particular solutions of the problem.

3. This is the summary of a memoir shortly to appear in the *Acta Mathematica*. The main results which are there announced and discussed are the two following: (P denotes the body of negligeable mass; S, T the two centers rotating uniformly.) 1° Despite the Newtonian law which makes the force infinite at the points S and T, it is possible by elementary transformations to remove all analytic singularities and to represent analytically all the arcs of the path, which fall within suitable regions about S and T. 2° In many cases a simple inspection of the initial conditions enables us to foresee that the distances of P from S and T will remain within fixed limits. The question of stability as stated by Poincaré is therefore answered by Levi-Civita in the affirmative.

6. For a long time, only two sorts of questions were proposed in connection with the determination of an integral of a linear partial differential equation — the problem of Dirichlet or the analogous problem of Bjerknes and Neumann for Laplace's equation; and the problem of Cauchy for equations with real characteristics. These two classes of problems are far from being the only ones or even the most important ones which come up in applications to mechanics and physics. Indeed in general, whether it be the hyperbolic or elliptic case, one has to do with *mixed* problems, that is to say the boundary is divided into several regions in which the initial conditions are of different kinds. These *mixed* problems are in particular of two sorts. In the elliptic case there is given either the value of the unknown function or that of one of its derivatives

according to the regions considered. In the hyperbolic case there is given the value of the unknown function and of its derivative (as in the problem of Cauchy) on a part of the boundary and the value of the unknown function on the rest. This last mixed problem is fundamentally different from that of Cauchy and from various points of view resembles that of Dirichlet and the other problem of the elliptic case. It depends in an essential way upon the forms of the boundary — a thing which does not happen in the problem of Cauchy.

7. On the basis of the simple and rigorous presentation of the field of a moving electron which Sommerfeld published in the *Göttinger Nachrichten*, 1904, the force and moment opposing the motion can be calculated in a general manner. When these forces are not equal to zero, they give the law of motion. In particular the rotations around a fixed center were discussed. The law of motion appears here as an integral equation and can be transformed into a differential equation of infinite order. There is an infinite series of free fundamental vibrations. In the case of superficial charges these are undamped and are determined by the equation $\tan \gamma = \gamma$. The energy pulsates freely in the interior of the electron and is latent relative to the exterior. If it be imagined that the atom of radium carries out such free vibrations and that its energy of rotation is in some way transformed into energy of translation, then an angular amplitude of small size suffices to explain the observed velocities of the β-rays of radium.

8. This communication forms a sequel to the paper which was read in 1894 at the meeting of the French Association and in which progressive and regressive multiplications were explained by statics for the plane. An idea of the method may be obtained from the following example. Let Σf denote a series of forces in equilibrium and let their rotors be cut by any plane in the points ΣA. The force f at A may be resolved into n normal to and t in the plane. Then ΣnA is a system of masses in equilibrium. Hence the product of a line and a plane may be taken as a point with an easily defined result. One simple geometric result may be noticed. Let four generators of the same system on a hyperboloid of one sheet be cut in the points A, B, C, D by one of a system of parallel planes. Then the *ratios* of the areas of the triangles ABC, BCD, etc.,

are constant, although the configuration does not remain similar to itself.

9. Weber's first note was concerned with the theory of vibrating strings and sought to explain the fact that although the variables enter in a symmetric manner in the equation

$$\partial^2 u/\partial x^2 - \partial^2 u/\partial y^2 = 0,$$

yet the boundary conditions for $x = 0$ and for $y = 0$ behave differently. (Riemann-Weber, Die partiellen Differentialgleichungen II, §90.) The second note considered Riemann's theory of finite vibrations in air and discussed the question of the connection of the discontinuities discovered by Riemann with the principle of conservation of energy. (*loc. cit.*, §§179–180.)

12. Finsterwalder spoke of exposures in which the photographic viewing-points were not accurately determined. Two exposures in which the constants of the apparatus were known, and which were properly oriented toward the vertical, suffice for reconstructing to scale the object photographed and the viewing-points. The rigorous solution of the problem leads to an equation of the sixth degree for which the speaker gave convenient graphical methods of approximation. To carry out the solution an auxiliary projection of the object was introduced. In this the net of n viewing-points in space is represented by a net of $n(n-1)/2$ points in the plane, lying three by three on $n(n-1)(n-2)/6$ straight lines. In practical work it is better to replace this net by a polar reciprocal net of lines. The method was tried on a practical example in which from eight exposures at six unknown viewing-points the alpine end of the Val di Genova was reconstructed on a scale of 1: 10,000.

13. Although in the motions of fluids arising in technical work the friction in the interior of the fluid plays a very small rôle, the theory of frictionless fluids agrees but poorly with experiment. If a small instead of zero value of the coefficient of friction be assumed its effect becomes noticeable when without it a finite difference of velocity would arise — as at the wall of a rigid body. It is possible to approximate very closely to the actual condition, if the effect of friction be taken account of to the first order in a thin layer along the wall, while in the free fluid no friction be granted. The appearance of surfaces of separation (Helmholtz's vortical sur-

faces) at continuously curved walls becomes recognized as due to friction. Prandtl showed with the stereopticon some experiments and explained them with the aid of the theory.

14. Kempe's paper was for the purpose of demonstrating a link work invented by him, and shown at the exposition held in connection with the congress. The chief object was to draw continuously the curves which he published in 1898 for dividing any possible angle into $2^n - 1$ and $2^n + 1$ parts, and which once drawn require only *one* transversal to find the $1/(2^n - 1)$ and $1/(2^n + 1)$ parts of any angle placed in the center of the diagram — that is to say, to find the complete division of an angle into any equal parts. (*Mémoires de la Société royale des sciences de Liège*, volume 20, 1898; *Zeitschrift für Mathematik und Physik*, volume 49, 1903.)

15. The calculating machine invented by Leibniz and constructed during his life was shown to the meeting. In essentials it agreed with the modern arithmometer of Thomas. The mechanism for carrying the tens is incomplete to such an extent that several successive operations of this sort cannot be accomplished. Thus in adding 1 to 9999 the machine gives 9900 instead of 10000. This inconvenience Leibniz alleviated by introducing a signal which showed the failure to carry the tens. The operator was thus reminded to carry the tens with another stroke of his finger.

16. Schilling divided his communication into two parts: first the different methods of projection for mathematical instruction; second the manifold ways of using the stereopticon for didactic purposes — both amply illustrated with lantern slides. As for the first, he described the preparation of slides, the marking of lines with colored inks, etc. As regards the second point, he spoke in detail of the different opportunities for using the stereopticon for geometry, for references to literature, for collecting of formulas, etc.

Section V. *History.*

Presidents: M. Cantor and Stäckel. Chairmen: Zeuthen, P. Tannery, von Braunmühl, and Loria.

(1) Cantor (Heidelberg): "Einführung in die Geschichte der Mathematik; Hinweis auf neue Resultate." †

(2) TANNERY, P. (Paris): "Résumé de la correspondance de Florimona, Debeaune et de Descartes."

(3) DICKSTEIN (Warsaw): "Wronski als Mathematiker."

(4) SIMON (Strassburg): "Ueber die Mathematik der Egypter." †

(5) ZEUTHEN (Copenhagen): "Gebrauch und Misbrauch historischer Benennungen in der Mathematik." †

(6) SCHLESINGER (Clausenburg): "Bericht über die Herausgabe der gesammelten Werke von L. Fuchs und Ueberreichung des I. Bandes."

(7) ENESTRÖM (Stockholm): "Welcher Platz gebührt der Geschichte in einer Encyclopädie der mathematischen Wissenschaften?" †

(8) VON BRAUNMÜHL (Munich): "Zur Geschichte der Differentialgleichungen." †

(9) SUTER (Zürich): "Zur Geschichte der Mathematik bei den Indern und Arabern." †

(10) LORIA (Genoa): Sur l'histoire de la géométrie analytique." †

(11) VAILATI (Como): "Intorno al significato della differenza tra gl'assiomi ed i postulati nella geometria greca." †

1. Cantor emphasized the fact that the investigations in the history of mathematics were capable of development on two sides — toward the future, because what a few decades ago was the present is now history — toward the past, because the philologists and archeologists are discovering new material. Examples show how the history of mathematics has thereby been changed, and also how the philologist and archæologist have grounds for taking account of the investigations of mathematicians. Thus the history of mathematics has more and more acquired the rank of a branch of science justified in itself and taught in special courses at many places of higher instruction. To it other branches of knowledge are connected, even a general history of science — toward which the speaker adopted a rather doubting attitude.

4. Simon protested against a point of view adopted even in the second edition of Cantor's great historical work. He held that the papyri could not be wholly trusted as adequately representing the knowledge of the Egyptians. He pointed out that the majority of the papyri should be regarded as pupils'

note-books, often transcribed by persons of moderate attainments at best. This he corroborated with examples. Further examples show that the Egyptians were in possession of a number of mathematical theories, the principles of similarity, the elements of descriptive geometry, etc.

5. As an example of nomenclature which, despite its departure from strict historical accuracy, should be respected as sufficiently descriptive, Zeuthen mentioned "the Pythagorean theorem" and "Napierian logarithms." On the other hand he disapproved of the terminology adopted by some who at present are interested in the foundations of geometry. They apply the term "theorem of Pascal" merely to a special case which in no wise interested Pascal; they use "axiom" in the sense in which Euclid applied the term "postulate"; they call "axiom of Archimedes" a postulate which is probably due to Eudoxus and which at all events was well known to Aristotle and Euclid (V, def. 4). In fact Euclid expressly develops in his first four books that which is at present called "non-archimedean geometry."

7. Eneström first remarked that by the history of mathematics he understood a useful history of the development of the mathematical theory, and gave to the word encyclopedia the same meaning as the founders of the mathematical encyclopedia now appearing. He then investigated the aim of such an encyclopedia and came to the conclusion that not only should historical notices be inserted in the special articles but that there should be added a general exposition of the historical development of the discipline. Next he mentioned what historical questions should find place in such an exposition and emphasized that the arrangement which he spoke of could be regarded really as a programme for an extension of the original plan of the encyclopedia.

8. Newton in his Methodus fluxionum treated differential equations of the form $F(\dot{x}, \dot{y}, \dot{z}, \cdots, x, y, z) = 0$, for which he gave as a sole example $z\dot{x} - \dot{z} + x\dot{y} = 0$. His solution of this so-called pfaffian differential equation shows that he thoroughly understood the character of this sort of equations, and only the lack of a functional notation prevented him from giving the general representation of both integrals. Moreover he evidently

gave only this one example because it was beyond his power to give to the solution a geometric interpretation — Monge (1784) was the first to succeed in so doing. The first *partial* differential equation is found in Euler, and came in his way when he sought the multiplier of the differential equation $dz + Pdx = 0$. From 1762 on Euler and d'Alembert treated partial differential equations systematically. The most general equation solved in this year by Euler is $z = \phi(p, q)$. D'Alembert, however, gave a method for solving the linear partial differential equation of the first order

$$\frac{\partial z}{\partial x} + \phi(x, y)\frac{\partial z}{\partial y} + z\psi(x, y) + \chi(x, y) = 0,$$

and Laplace in 1773, that is a year before Lagrange, integrated the equation

$$\frac{\partial z}{\partial x} + \phi(x, y)\frac{\partial z}{\partial y} + \psi(x, y, z) = 0.$$

9. Suter first treated the two remarkable formulas for polygons found in Bhāskara's Līlāvatī (Cantor, Vorlesungen, 2d edition, volume 1, page 618), and sought to show how the Hindus could have arrived at these formulas. Secondly, he gave grounds to establish the probability that the "liber augmenti et dimensionis," published by Libri in his Histoire des sciences mathématiques en Italie (volume 1, pages 304–368), was written by Abū Kāmil Shoǵā' ben Aslam (al. Abraham), an Arabian mathematician living in Egypt about the year 900.

10. Loria began with an aperçu of Baltzer's Analytic Geometry, and asked how that author could say that this branch of mathematics is a grand-niece of Greek geometry. He remarked that this relationship becomes evident if one turns to the Geometry of Descartes, of which he pointed out the resemblance rather to the works of Apollonius than to the modern treatises on analytic geometry, which resemble much more closely the Isagoge of Fermat. It is known, however, that the ideas of this great man have become appreciated only in recent years, whereas those of Descartes spread and developed in an astonishing manner. Loria specially cited among the continuers of Descartes, Roberval, de la Hire, and the marquis de l'Hôpital, and he then stopped to mention the contributions

which Newton made to this doctrine. The appearance of Euler was the signal for the commencement of a new era in analytic geometry as in the other branches of pure and applied mathematics. Despite this, despite the great work of Cramer, it was Lagrange who was destined to place geometry on a true analytic basis. His ideas, combined with those which rendered Monge immortal, appreciated and completely developed by Lacroix and Biot, finally transformed a chapter on the introduction to infinitesimal geometry into an independant branch of pure mathematics. The period of Lagrange, Monge, and Lacroix has therefore an extraordinary importance comparable to that of the period of Descartes and Fermat. Loria said that it is these periods which should have the especial attention of whosoever would write a history of analytic geometry; and he closed by expressing the hope that mathematical literature might be enriched by such a history, of which the necessity could not but be admitted by all.

11. In a remarkable passage of his commentary on Euclid's elements, Proclus alludes to three different significations which had been attributed by his predecessors to the distinction between axioms and postulates. According to one, the postulates were the propositions affirming that a certain construction can be effected, while the axioms were the propositions (undemonstrated) affirming that a construction, supposed or proved to be possible, could not be effected without the resulting figures enjoying or not enjoying some property or other (which is not implied in its definition). According to the second, the properties (undemonstrated) relative to relations which can subsist between any quantities (geometric or not) are called axioms, while the propositions (undemonstrated) of which the import does not extend beyond the field of geometry or spatial relations are called postulates. According to the third, axioms are those properties, the truth of which follows from the very definition of the terms which figure in it, while postulates are the other undemonstrated propositions which are not in this category and which therefore may be denied without falling into contradiction. It is interesting to note that each of these three distinctions is related to criteria, the importance of which is becoming emphasized by recent researches on the logic of mathematics. The first, in part, is relative to the distinction between particular propositions and general propositions in the

sense of Leibniz. The second is relative to the distinction between pure and applied mathematics as stated by Russell in his Principles of Mathematics. Finally, the third agrees with the distinction which modern philosophy expresses by setting in contrast synthetic and analytic judgments. It is important to bring out the fact that in none of the three distinctions mentioned by Proclus is it a question of any contrast (between axioms and postulates) relative to their different "degrees of evidence."

Section VI. *Pedagogy.*

Presidents: Schubert and Treutlein. Chairmen: Lampe, Greenhill, Fehr, Schotten, and Gubler.

(1) KLEIN (Göttingen): Ueber eine zeitgemässe Umgestaltung des mathematischen Unterrichts an den höheren Schulen."

(2) SCHUBERT (Hamburg): Elementare Berechnung der Logarithmen."

(3) BUFFA (Nocera): "Primo studio della geometria piana."

(4) GREENHILL (London): "Exercises in practical mathematics."

(5) GREENHILL (London): "Teaching of mathematics by familiar applications on a large scale." †

(6) GUTZMER (Jena): "Ueber die auf die Anwendungen gerichteten Bestrebungen im mathematischen Unterricht an den deutschen Universitäten." †

(7) LORIA (Genoa): "Sur l'enseignement des mathématiques en Italie." †

(8) VERONESE (Padua): "La Laguna di Venezia" (by title).

(9) FEHR (Geneva): "Enquête de la revue internationale, L'Enseignement Mathématique, sur la méthode de travail des mathématiciens." †

(10) STÄCKEL (Kiel): "Ueber die Notwendigkeit regelmässiger Vorlesungen über elementare Mathematik an den Universitäten." †

(11) FRICKE (Braunschweig): "Ueber den mathematischen Unterricht an den technischen Hochschulen in Deutschland." †

(12) ANDRADE (Besançon): "L'enseignment mathématique aux écoles professionnelles et les mathématiques de l'ingénieur."

(13) SCHOTTEN (Halle): "Welche Aufgabe hat der mathematische Unterricht auf den deutschen Schulen und wie passen die Lehrpläne zu dieser Aufgabe?" †

(14) SIMON (Strassburg): "Ueber Unterricht in der sphärischen Trigonometrie." †

(15) THIEME (Posen): "Wirkung der wissenschaftlichen Ergebnisse auf den Unterricht in der elementaren Mathematik."†

(16) SOUREK (Sofia): "Ueber den mathematischen Unterricht in Bulgarien."

(17) MEYER (Königsberg): "Ueber das Wesen mathematischer Beweise."

(18) FINSTERBUSCH (Zwickau): "Ueber eine neue und vor allem einheitliche Methode, die Rauminhalte der Körper zu bestimmen, deren Querschnittsfunktion den dritten Grad der Höhe nicht übersteigt."

(19) BRÜCKNER (Bautzen): "Ueber die diskontinuierlichen und nicht-konvexen gleicheckig-gleichflächigen Polyeder." †

1–4. These papers were distributed in printed form.

5. A collection of exercises was distributed showing the method of instruction in dynamics in the advanced class of the Ordnance College at Woolwich. Attention was directed to the concrete practical nature of the interest, as time does not allow a complete theoretical study. The use of a mathematical workshop was recommended as a useful stimulus. In this the student verifies the laws of energy and momentum with homely experiments. In our present laboratories we are really following much earlier methods. Galileo's swinging lamp, for example, suggested to him not mechanical application of the pendulum, but a simple means of determining equal intervals of time.

6. In the first part of his communication Gutzmer spoke of the rise of the tendency toward applications in university instruction. In the second part he took certain exceptions to the theses of Stäckel (*Jahresbericht der deutschen Mathematiker-Vereinigung*, volume 13, pages 340–1). While agreeing in the main with Stäckel, he wished to emphasize the differences: 1° that every student of mathematics should pursue descriptive geometry, as in southern Germany, 2° that in pure mathematics the requirements for the examination *pro facultate docendi* be not lowered from the present standard, 3° that in applied mathematics more attention be paid to technical mechanics than is recommended by Stäckel, in order that the mathematicians might better understand the problems of technical science and be better prepared for teaching in schools of

technology, 4° that the requirements in applied mathematics and physics for the doctorate be as accepted at Göttingen and Jena. In the third part he took up the pedagogic development of the teacher. The scientific development (even in a pedagogic way) should be the task of the universities, but the introduction to the practice of teaching could best be obtained from experienced school teachers.

7. Loria commenced by remarking that the geometric traditions in Italy were frankly Greek, at all events they were at most partially weakened by the successive governments which followed each other. It was Cremona who proposed and brought to success the reëstablishment of instruction on a good basis by suggesting to the Italian government to force, in the classical schools, a return to the pure sources of Greek geometry. This was moderated after a few years by demanding merely that instruction follow the method and not necessarily the text of Euclid. This movement gave rise to a series of good modern treatises such as those of Sannia, d'Ovidio, and Faifofer; while eminent scientists, as Paolis, Veronese, Enriques, and Amaldi, turned their attention to the foundations of geometry. The results they obtained, the discussions aroused by their propositions, the attempts which their students made to use these in instruction, infused the body of instructors with new life. Of this a striking proof is found in the association Mathesis, allying the instructors of the secondary schools of Italy for the purpose of "making use of the progress of science for the purposes of the schools." Loria next examined two questions: 1° Is it useful from a didactic point of view to abandon the euclidean method and to treat conjointly the geometry of plane and space; 2° Is there any way of increasing the profit which the student should get from secondary instruction? Finally the speaker touched on the Italian technical schools and mentioned the changes which sooner or later must take place in their curricula. (The paper will shortly appear in *l'Enseignement Mathématique.*)

8. Fehr presented the series of questions which the *Enseignement Mathématique* submits to mathematicians to ascertain their method of work. The object of the inquiry is to collect the opinions of mathematicians on the methods of their work in order that from the mass of replies there may be sifted out a

number of points of information and advice which may be useful not only to young mathematicians but to mathematical instruction in general. The inquiry contains thirty questions of a psychologic and physiologic nature. It is in fact interesting to know not merely the habit of work but the general hygienic habits of mathematicians, and the means which they judge best fitted to facilitate intellectual work. It is to be hoped that all mathematicians will lend their collaboration in the form of replies to these questions in order that the results of the inquiry may well represent the general opinion. (The series of questions with blanks for reply may be obtained of Professor H. Fehr, 19 rue Gevray, Geneva, Switzerland.)

10. The practical pedagogic training of the candidates for positions as teachers does not belong to the universities; but it is desirable that the present chasm still existing between university science and school instruction be filled in. Besides vacation and information courses in which the universities are called to coöperate there should be for this purpose in the regular curriculum itself 1) elaborate consideration of the applications, 2) lectures in which elementary mathematics is treated from a higher standpoint. Such lectures on elementary mathematics 1) would be easily fitted in to the present scheme of university instruction and indeed should find their place at its close, 2) would notably aid and support the future pedagogic development of the candidates, 3) would also contribute to keep the teachers of mathematics in living contact with their science. To accomplish this aim these lectures should awake an interest for the elements and give a deeper insight into them; in this the scientific and mathematical, and the historical and literary import alike demand consideration.

11. Fricke spoke of the differences which arose about ten years ago between the teachers of the technical subjects and teachers of mathematics in technological schools. He then entered into opposition with Klein, regarding the latter's proposition to introduce the elements of differential and integral calculus and of the theory of functions into the curricula of the higher secondary schools. In closing he warmly recommended the striving on the part of the technical schools to prepare teachers of mathematics and technology for the gymnasia and secondary schools.

13. Schotten, without at all contesting the formal value of the instruction, wished that the material side be better attended to. He insisted on the difference between the problems given out as exercises and the applications. In a reform, however, we should not pay attention merely to mathematical and physical science; the entire field must be considered. Plans had been proposed on different sides, especially by those interested in hygiene and in German nationalism, for a new arrangement of higher instruction. It seems as if a new ideal of education was coming in. The speaker asked that in outlining the curricula every tendency to overload be abandoned. Rather keep to the motto "hungry better than overfed." The aim of mathematical instruction should be to foster a pleasure in reflective thinking and individual work. To effect this several changes were suggested, the present curricula being unhappily chosen. For example, let the matter taught be collected in separate grades, and let the unfortunate effect of the hurry-to-get-there ("Hetzbetrieb") be removed.

14. Simon first remarked that problems which lead to a quadratic equation may be solved without the introduction of imaginaries, whereas the equation of the third degree requires necessarily imaginary numbers. It is therefore no mere chance that Cardan was the first to work with complex numbers. As a consequence the speaker inferred that the curricula of the secondary schools should either contain no imaginaries or introduce them in connection with the cubic equation. Next he passed to the consideration of spherical trigonometry. He recommended a number of changes in the method of instruction. In short, these amounted to nothing but a return to the method followed by Euler.

15. The scientific investigations which may be of use in elementary mathematics were divided by Thieme into three categories: First, science has given to the elements a logical foundation; second, a certain number of recent researches bring more general questions to the domain of elementary mathematics and thus render possible a deeper insight into the connections between the branches; third, of late years certain parts of elementary mathematics such as the geometry of the triangle have made great advances. In dealing with the question in how far these investigations can be used in the instruction of ele-

mentary mathematics account must be taken of the time alloted to instruction, of the comprehension of the student, of the present state of instruction, and the extent to which these researches have spread among the teachers. Many changes which could be heartily wished for cannot be made owing to the limits imposed by such conditions as those just stated — for instance, the ability to comprehend on the part of the students in the secondary schools may not be sufficient. But in many points the acquisitions of science should be more taken account of than is the case. Science cannot turn over to the beginner, whether in arithmetic or in geometry, a complete and rigorously logical system, instruction must proceed along the way of intuition. But this only within certain limits. The intuitive idea of positive and negative number as profit and loss should be used only in the first grade, not in the second or third. Moreover the conceptions of equal, greater, less, sum, product, power are defined only for positive integers; if they be used for negative, rational, irrational and imaginary numbers, one should define them for such numbers. Hence the equations

$$(-a)(-b) = +ab, \quad a^{-n} = \frac{1}{a^n},$$

$$a + bi = c + di \text{ if } a = b \text{ and } c = d,$$

should be regarded not as theorems but as definitions. Even in the lower grades the conceptions should be enunciated correctly and their domain of validity given, although their logical development may be allowed to remain in the background. In the upper grades the whole logical development of arithmetical conceptions and laws in a connected manner should be given to the students. Here the irrational and imaginary numbers should be introduced with care. Infinite series and a short introduction to differential and integral calculus should here find a place. In geometry a strict, complete logical system is beyond the student's power of conception, but many things that science has shown to be false may be modified. At first one may leave aside definitions of surfaces, curves, straight lines, and points. All the definitions in use are inaccurate. The majority of the definitions of angle are equally bad. The many inaccurate proofs should be given up. To be sure, in the lower grades we cannot exact the use of the complete systems of axioms as given by Pasch, Veronese, Peano, and Hilbert; we

must be satisfied with the axioms of connection and of parallels. In the upper grades, however, the student should learn what a rigorous and connected logical system geometry really is even if he does not carry the system through in detail. The speaker went on further to explain how and why the elements of projective geometry and nomography should be introduced. He closed with a short résumé of the actual state of instruction in Germany, and cited in behalf of his opinions the Arithmetik of Stolz and Gmeiner, the Encyclopedia of Elementary Mathematics by Weber and Wellstein, and the geometric treatises of Pasch, Hilbert, Veronese, and Ingrami.

19. Brückner, after some introductory remarks on the historical development of the problem of determining the equiangular, equisuperficial polyhedra — a problem solved for convex polyhedra by E. Hess — proceeded to give a résumé of the results he had found concerning the non-convex and discontinuous polyhedra of this type. Beside the sphenoids of the octahedral and icosahedral systems (7 and 5 groups, respectively) he went into the stephanoids of these two systems (3 and 11 combinations, respectively). The models of these discontinuous "null-polyhedra," and those of the continuous non-convex polyhedra with positive content, and those of Möbius's polyhedra of the icosahedral system were exhibited. (The complete results will be published in the *Nova Acta* of the Leopold Academy.)

EDWIN BIDWELL WILSON.

YALE UNIVERSITY.

THE BRESLAU MEETING OF THE DEUTSCHE MATHEMATIKER-VEREINIGUNG.

The annual meeting of the Deutsche Mathematiker-Vereinigung, forming a part, as usual, of the annual meeting of German scientists and physicians, was held this year at Breslau, in Silesia.

The first forenoon, Monday, September 19, was devoted to a general meeting of the entire association. In the afternoon of the same day the sessions of the different sections began. The convention consisted of 13 scientific and 17 medical sections,

and not less than 650 papers and demonstrations were presented during the week.

In the mathematical section the attendance was very light, and but few papers were offered. It was evident that the Heidelberg congress, a few weeks previous, had practically cleared the field.

Although pure mathematics were somewhat neglected, this was offset by an unusual activity along pedagogical lines. On Thursday forenoon a general meeting was devoted to the topic: "Scientific and mathematical instruction in the higher secondary schools." The presence of representatives of the government and of teachers' associations gave evidence of a strong public interest in this subject. Professor A. Fricke (Bremen) opened the discussion with a paper on: "The present condition of instruction in natural science and mathematics in the schools of higher learning." He emphasized the fact that the German gymnasium is too much inclined along medieval lines; that is, the philological-historical side receives too much attention, while scientific instruction is not modified in accordance with new developments.

Professor Felix Klein then spoke on "New tendencies in the mathematical-physical field." He outlined the steps that had been taken to improve instruction in mathematics and physics, and then took up the methods used at present in teaching mathematics. He attributed the general aversion to mathematics in the gymnasia, not to the subject matter, but to the manner of presentation, and suggested as a remedy vacation courses in pedagogy for teachers, and a system whereby a teacher may obtain regular leave of absence in order to study and keep abreast of the times. Because of the position mathematics has assumed within the last century, as the basis for the exact sciences, he pleaded for an increase in the number of hours of instruction in mathematics at the expense of the ancient languages.

Other papers, bearing on instruction in biology and on school hygiene, were presented. A very animated general discussion followed with the result that a commission of twelve was appointed to consider the matter, and to report at the next meeting.

While the days were devoted to earnest work, the evenings were given up to social enjoyment. Monday evening occurred a presentation of the "Flying Dutchman" in the Stadt-theater; Tuesday evening, a banquet; Wednesday, small gatherings at

different points; Thursday, supper in the Zoological garden followed by a boat ride on the Oder; and Friday evening, a banquet, fireworks, etc., after which the convention closed with a hearty "Auf Wiedersehen."

The following is a partial list of papers presented before the Mathematical section:

(1) LAMPE (Berlin-Charlottenburg): "Some examples of the exercises in the integral calculus at the Charlottenburg technical school."

(2) GUTZMER (Jena): "The theory of linear homogeneous differential equations."

(3) KOWALEWSKI (Greifswald): "On a generalization of the second mean value theorem of the integral calculus."

(4) STURM (Breslau): "On the Cremona transformations in which to the planes of the one space correspond general surfaces of the third order in the other space."

(5) LANDSBERG (Heidelberg): "On the analogies between the theories of algebraic numbers and algebraic functions."

(6) STEINITZ (Charlottenburg): "Collinear depictions of trigonal polyhedrons, and the analysis situs in projective space."

(7) LUDWIG (Karlsruhe): "On the theory of contact transformations of the circles on a sphere."

(8) GUTZMER (Jena): "On the theory of adjoined differential equations."

A brief abstract of each paper, so far as obtainable, follows:

1. In this paper is described the scope and character of the work in the Charlottenburg technical school during the first academic year, as it has been organized by Professor Lampe. As typical exercises, the following may be taken:

(*a*) Quadrature of the curves $(x/a)^p + (y/b)^q = 1$ for $x > 0$, $y > 0$ from $x = 0$ to $x = a$ and for given values of p and q, correct to a specified number of decimal places.

(*b*) Determination of the volumes inclosed by certain curved surfaces whose equations are first to be found; *e. g.*, given in polar coördinates r, ϕ the equation of the curve $r^2 = a^2 \cos^2 \phi + b^2 \sin^2 \phi$, a variable parabola has its vertex at a fixed point of the perpendicular to the plane of the curve through its centre; this perpendicular is the principal axis, and the parabola passes through the extremities of a diameter of the curve. The volume between the surface generated by the parabola and the given curve is to be found. Generalization of this exercise to other curves.

(*c*) The conic $r = \rho/(1 + e \cos \varphi)$ is given. On each radius vector r, as diameter, a circle is described in a plane perpendicular to the plane of the conic. Find the volume of the surface so generated in the several cases $e < 1$, $e = 1$, $e > 1$.

2. Professor Gutzmer's first paper discussed the derivation of a method for determining the reciprocal differential equation and its employment in certain special cases in which it is particularly useful.

3–4. The papers of Professor Kowalewski and Professor Sturm will appear in the *Jahresbericht*.

5. Professor Landsberg compared the theory of algebraic numbers and of algebraic functions and called attention to certain analogies which these subjects furnish in their present state of development. His paper will appear in full in the *Jahresbericht*.

6. This paper treats of the depiction of a polyhedron whose faces are triangles upon a second polyhedron, whose faces are triangles, in such a way that a one-to-one correspondence exists between the vertices and between the faces of the two polyhedrons. Let $a, b, c, \cdots$ be the vertices of the first polyhedron and $a', b', c' \cdots$ the corresponding vertices of the second. In order to depict one triangle abc on the corresponding triangle $a'b'c'$, it is only necessary to associate an arbitrary point in the first with an arbitrary point in the second. If we continue this process we come finally to a point on an edge ab which belongs to two triangles and can be represented on $a'b'$ in two ways. At this point the one-to-one correspondence fails, but it can be maintained if the projective representation of the edges is fixed. It is necessary in order to bring two edges in relation to select a point pair p, p'. This is analytically possible if the value of $a'p'/p'b' : ap/pb$ or the real logarithm is given. The last value changes sign with a change in direction. The selection of the projective relation is, then, not arbitrary. Geometrically the condition is, that the lines joining the midpoints of the sides of the triangle abc to the vertices $a'b'c'$ must pass through one point. Analytically, the condition is, that the sum of the logarithms belonging to any three sides of a triangle must be zero. We will call the logarithm belonging to any edge (ab) its integral value and denote it by $\int_a^b$. From the fore-

going we have then $(ab) = -(ba)$, $(ab) + (bc) + (ca) = 0$; as in the theory of abelian integrals of the first and second kinds, we conclude here that every complete integral whose path is a simply connected surface must vanish. One obtains here a group of ∞^{e-1} solutions (e denotes the number of vertices) when one gives arbitrary functional values $f(a)$, $f(b)$, $\cdots$ to each of the vertices a, b, c $\cdots$ and defines the integral values of the edges ab, $\cdots$ by means of the equation

$$\int_a^b = f(b) - f(a).$$

This last result does not change when the functional values of the vertices are increased by a constant quantity. In this manner only those solutions are exhausted whose complete integral is zero. The presence of moduli of periodicity in the case of integrals of the first and second kinds shows that still other solutions exist. In fact, in the case of a bilateral polyhedron there are in all $\infty^{(e-1)+z}$ solutions, where $z = k + 2 - e - f = 2p$. k = number of edges, e = number of vertices, f = number of surfaces, p = genus.

In the case of a unilateral polyhedron there are $\infty^{(e-1)+(z-1)}$ solutions. A close connection exists between this problem and certain investigations in the topology of projective space, which gives the following result. While in euclidean space corresponding points on two surfaces which in the sense of analysis situs are equal, *i. e.*, permit of a one-to-one depiction on each other, can be made to coincide, in projective space this is not the case. Each type of analysis situs, from this point of view, falls into 2^z subtypes.

7. A purely geometric investigation of the contact transformations of the circle on a sphere requires that one consider them as applied to the entire sphere and in their complete multiplicity. It is therefore necessary to start from very meager conditions, *i. e.*, a contact transformation L of the circle must associate with every point on the sphere a circle and transform every circle into two circles (which may coincide). The same must be true for the inverse transformation L^{-1}. It appears that this transformation contains ten constants. Lie had shown by a different method that all contact transformations of a circle on a sphere form a ten-element group. The author shows that these two methods lead to the same group L.

8. The second paper by Professor Gutzmer consists of a short report on some investigations and results related to a previous paper by him under the title: "Theory of adjoined differential equations," Halle, 1896.

The next meeting of the Vereinigung will be held at Meran, in the Tyrol, in September, 1905, under the presidency of Professor F. Klein.

R. E. WILSON.

GÖTTINGEN, GERMANY,
November, 1904.

THE CONSTRUCTION OF CONICS UNDER GIVEN CONDITIONS.

BY PROFESSOR M. W. HASKELL.

(Read before the San Francisco Section of the American Mathematical Society, April 30, 1904.)

UNDER the above title are grouped a few notes on special aspects of the subject, with no pretense at an exhaustive treatment. While certain of the results are not new, it is believed that the method by which they are derived will be of some interest. Homogeneous coördinates are used exclusively; following Casey (Analytical Geometry, page 70), a quadrangle or quadrilateral whose diagonal triangle is the triangle of reference is designated as a "standard" quadrangle or quadrilateral. The coördinates of the vertices of a standard quadrangle, or of the sides of a standard quadrilateral, are then $\pm\kappa_1 : \pm\kappa_2 : \pm\kappa_3$, and they may be projected into $\pm 1 : \pm 1 : \pm 1$ if desired. It is to be noticed that the complete quadrangle (quadrilateral) is fully determined and can be easily constructed, when the diagonal triangle and any one vertex (side) are given.

§ 1. *Conics Defined by Five Points or by Five Tangents.*

THEOREM I. *The eight vertices of two standard quadrangles lie on one and the same conic.*

For, let the vertices be respectively $\pm\kappa_1 : \pm\kappa_2 : \pm\kappa_3$ and $\pm\lambda_1 : \pm\lambda_2 : \pm\lambda_3$; it is clear that they all lie on the conic whose equation is

$$\begin{vmatrix} x_1^2 & x_2^2 & x_3^2 \\ \kappa_1^2 & \kappa_2^2 & \kappa_3^2 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{vmatrix} = 0.$$

This theorem affords the following simple construction by points of a conic of which five points are given.

CONSTRUCTION: Construct the complete quadrangle of any four of the given points, and regard its diagonal triangle as the triangle of reference. Then complete the standard quadrangle of which the fifth given point is one vertex. The remaining three vertices lie on the conic. The five given points lead us thus immediately to fifteen new points, and the construction can be carried on indefinitely.

I will omit here, and in general in what follows, all theorems which can be derived from those given merely by the principle of duality.

THEOREM II (LEMMA). *The lines joining corresponding vertices of two standard quadrangles form a standard quadrilateral.*

For, if the vertices of the given quadrangles be respectively $\pm\kappa_1 : \pm\kappa_2 : \pm\kappa_3$ and $\pm\lambda_1 : \pm\lambda_2 : \pm\lambda_3$, the coördinates of the lines joining corresponding vertices will be

$$\pm(\kappa_2\lambda_3 - \kappa_3\lambda_2) : \pm(\kappa_3\lambda_1 - \kappa_1\lambda_3) : \pm\kappa_1\lambda_2 - \kappa_2\lambda_1).$$

Consider now two standard quadrangles $ABCD$, $A'B'C'D'$ and the conic upon which they lie. If A' be made to approach A, $B'C'D'$ will approach BCD respectively, the chords AA', BB', CC', DD' approaching the tangents at A, B, C, D. Hence

THEOREM III. *The tangents at any four points of a conic form a quadrilateral which is standard with reference to the diagonal triangle of the quadrangle of those four points.*

From this theorem and from the dual of Theorem I we have immediately

THEOREM IV. *Let the four intersections of two conics form a standard quadrangle. Then the two quadrilaterals formed by their tangents are standard, and therefore envelop a conic.*

Similarly, the eight points of contact of the four common tangents to two conics lie on a conic.

THEOREM V. *The quadrilateral of the four common tangents to two conics has the same diagonal triangle as the quadrangle of their intersections.*

For, let PP' be one of these common tangents, P and P' being the respective points of contact. Then, if Q, R, S; Q', R', S', respectively be the remaining vertices of the quadrangles determined by P and by P', QQ', RR' and SS' will be the remaining sides of the quadrilateral determined by PP'. Q, R, S will lie on the first conic; Q', R', S' on the second; and QQ', RR', SS' will be tangent to both.

Moreover, the conic through the eight points of contact of the common tangents, and the conic touched by the eight tangents at the intersections of two conics, each have the triangle of reference as a self-polar triangle.

§ 2. *Harmonic Ranges on a Conic. Conics through Four given Points and Tangent to a given Line.*

THEOREM VI.—*Let A_1, A_2, A_3, A_4, P be five points on a conic. Let A_{ik} be the harmonic conjugate of P with respect to A_i and A_k. Then the lines joining A_{12} and A_{34}, A_{13} and A_{24}, A_{14} and A_{23} are concurrent with the tangent at P.*

Let the coördinates of the five points be

$$A_1(1:1:1),\quad A_2(-1:1:1),\quad A_3(1:-1:1),\quad A_4(1:1:-1),$$
$$P(\lambda:\mu:\nu).$$

It is not difficult to show that the equations of the three lines in question are

$$A_{12}A_{34}\quad \lambda(-2\lambda^2+\mu^2+\nu^2)x_1-\mu(\nu^2-\lambda^2)x_2+\nu(\lambda^2-\mu^2)x_3=0$$
$$A_{13}A_{24}\quad \lambda(\mu^2-\nu^2)x_1+\mu(\lambda^2-2\mu^2+\nu^2)x_2-\nu(\lambda^2-\mu^2)x_3=0$$
$$A_{14}A_{23}\quad -\lambda(\mu^2-\nu^2)x_1+\mu(\nu^2-\lambda^2)x_2+\nu(\lambda^2+\mu^2-2\nu^2)x_3=0$$

and these lines evidently meet in

$$Q\quad \left(\frac{1}{\lambda}:\frac{1}{\mu}:\frac{1}{\nu}\right)$$

Moreover, the equation of the tangent at P is

$$\lambda(\mu^2-\nu^2)x_1+\mu(\nu^2-\lambda^2)x_2+\nu(\lambda^2-\mu^2)x_3=0,$$

and this is satisfied by the coördinates of Q.

Now the points P and Q are interchanged by the involutory quadratic transformation

$$y_1 : y_2 : y_3 = \frac{1}{x_1} : \frac{1}{x_2} : \frac{1}{x_3},$$

of which the triangle of reference is the fundamental triangle and the points A_1, A_2, A_3, A_4 are the double points. Hence the line PQ is also tangent to the conic through A_1, A_2, A_3, A_4 and Q, and we are led to the following interpretation of the above transformation:

THEOREM VII. *The pairs of points which are interchanged by an involutory quadratic transformation of the plane are the points of contact of the tangents common to the pairs of conics of the pencil through the four double points of the transformation.*

Or again, they are the double points of the involutions defined on the lines of the plane by the pencil of conics through four points.

In general, under this transformation a straight line is converted into a conic through the vertices of the fundamental triangle; but this conic degenerates if the line in question passes through one of the vertices of the fundamental triangle, and in particular the sides of the complete quadrangle of the self-corresponding points are converted into themselves, corresponding point-pairs on one of these sides being harmonically conjugate with reference to the self-corresponding points of that side. For instance, the points $(1 + \lambda : 1 - \lambda : 1 + \lambda)$ and $(1 - \lambda : 1 + \lambda : 1 - \lambda)$ are evidently interchanged by the transformation, and they are harmonic conjugates with respect to $(1 : 1 : 1)$ and $(1 : -1 : 1)$. Hence follows

THEOREM VIII. *The conic into which a given line is converted by the quadratic transformation passes through the harmonic conjugates of the intersections of that line with the six sides of the complete quadrangle $A_1A_2A_3A_4$.*

A special case of this theorem is the conic passing through the middle points of the sides of a complete quadrangle and through the vertices of its diagonal triangle. It can also easily be shown that *the lines joining the pairs of such points which lie on opposite sides of the quadrangle are concurrent.*

These theorems lead us immediately to the construction of the two conics through four given points and tangent to a given line. For the respective points of contact are the intersections of the given line with the conic into which it is converted by

the quadratic transformation. *They are also of course the double points of the involution defined upon that line by its intersection with the three pairs of opposite sides of the complete quadrangle of the four given points.*

The question of the reality of the conics can be determined readily from the above. For the line $ax_1 + bx_2 + cx_3 = 0$ will intersect the conic $ax_2x_3 + bx_3x_1 + cx_1x_2 = 0$ in real points if

$$s(s-a)\,(s-b)\,(s-c) < 0$$

where $2s = a + b + c$, and the geometric interpretation of this result is that the given line must cut each pair of opposite sides of the complete quadrangle of the four points either in the segments upon which a vertex of the diagonal triangle lies, or else in those segments upon which it does not lie. If this condition is fulfilled for one pair of sides, it is also fulfilled for both the remaining pairs.

It may be worth while to remark here that the quadratic transformation is fully defined by the fundamental triangle and any pair of corresponding points P_1 and Q_1, the double points being determined by completing the standard quadrangles $P_1P_2P_3P_4$ and $Q_1Q_2Q_3Q_4$ and passing a conic through each of these sets of points and tangent to P_1Q_1; for the intersections of these two conics will be the double points required. This construction solves the problem of finding two conics, given their common self-polar triangle and the points of contact of one of their common tangents.

§ 3. *Conics with Two Points and Three Tangents given.*

Let the three tangents be taken as the triangle of reference and let the two points be $(a_1 : b_1 : c_1)$ and $(a_2 : b_2 : c_2)$. Then, writing

$$l = (\sqrt{b_1c_2} - \sqrt{b_2c_1})^2, \qquad l' = (\sqrt{b_1c_2} + \sqrt{b_2c_1})^2,$$
$$m = (\sqrt{c_1a_2} - \sqrt{c_2a_1})^2, \qquad m' = (\sqrt{c_1a_2} + \sqrt{c_2a_1})^2,$$
$$n = (\sqrt{a_1b_2} - \sqrt{a_2b_1})^2, \qquad n' = (\sqrt{a_1b_2} + \sqrt{a_2b_1})^2,$$

the equations of the four conics satisfying the given conditions will be

$$1) \qquad lu_2u_3 + mu_3u_1 + nu_1u_2 = 0,$$
$$2) \qquad lu_2u_3 + m'u_3u_1 + n'u_1u_2 = 0,$$

$$3) \quad l'u_2u_3 + mu_3u_1 + n'u_1u_2 = 0,$$
$$4) \quad l'u_2u_3 + m'u_3u_1 + nu_1u_2 = 0.$$

If the given points and lines are all real, then these conics are all real if no two of the given lines meet the line joining the two given points in points alternating with the given points; otherwise they are all imaginary.

Each pair of these conics will have a fourth common tangent, for example

$$t_{12}: \quad \frac{x_1}{mn' - m'n} + \frac{x_2}{l(n - n')} - \frac{x_3}{l(m - m')} = 0,$$

$$t_{34}: \quad \frac{x_1}{m'n' - mn} + \frac{x_2}{l'(n - n')} + \frac{x_3}{l(m - m')} = 0,$$

and it can easily be shown that the intersections of opposite pairs t_{12} and t_{34}, t_{13} and t_{24}, t_{14} and t_{23} all lie on the line joining the two given points.

The poles of the common chord. The four conics are fully defined as soon as the poles of the common chord are determined. Now, we have identically

$$lu_2u_3 + mu_3u_1 + nu_1u_2$$
$$\equiv (a_1u_1 + b_1u_2 + c_1u_3)(a_2u_1 + b_2u_2 + c_2u_3)$$
$$- (u_1\sqrt{a_1a_2} + u_2\sqrt{b_1b_2} + u_3\sqrt{c_1c_2})^2,$$

and similarly for the other conics. Hence the four poles in question are the points

$$\sqrt{a_1a_2} : \pm\sqrt{b_1b_2} : \pm\sqrt{c_1c_2}.$$

These points form a standard quadrangle and they are the double points of the quadratic transformation

$$y_1 : y_2 : y_3 = \frac{a_1a_2}{x_1} : \frac{b_1b_2}{x_2} : \frac{c_1c_2}{x_3},$$

which interchanges the two given points, the three given lines being the fundamental triangle.

UNIVERSITY OF CALIFORNIA,
April, 1904.

NOTES.

THE seventh regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY will be held at Stanford University on Saturday, February 25.

THE opening (January) number of volume 6 of the *Transactions* of the AMERICAN MATHEMATICAL SOCIETY contains the following papers: "On the linear transformations of a quadratic form into itself," by P. F. SMITH; "A set of postulates for real algebra, comprising postulates for a one-dimensional continuum and for the theory of groups," by E. V. HUNTINGTON; "On the primitive groups of class $3p$," by W. A. MANNING; "The minimum degree τ of resolvents for the p-section of the periods of hyperelliptic functions of four periods," by L. E. DICKSON; "Determination of all the groups of order 2^m which contain an odd number of cyclic subgroups of composite order," by G. A. MILLER; "On the coefficients in the quotient of two alternants," by E. D. ROE, JR.; "General theory of curves on ruled surfaces," by E. J. WILCZYNSKI; "Theory of plane curves in non-metrical analysis situs," by O. VEBLEN.

THE opening (January) number of volume 27 of the *American Journal of Mathematics* contains: "Some properties of a generalized hypergeometric function," by F. H. JACKSON; "Relation between real and complex groups with respect to their structure and continuity," by S. E. SLOCUM; "Determination of all the characteristic subgroups of any abelian group," by G. A. MILLER; "Collineations whose characteristic determinants have linear elementary divisors, with an application to quadratic forms," by A. B. COBLE; "Concerning certain elliptic modular functions of square rank," by J. A. MILLER; "Minors of axisymmetric determinants," by E. J. NANSON; "On the forms of sextic scrolls having a rectilinear directrix," by VIRGIL SNYDER. The number contains, as frontispiece of the volume, a portrait of Dr. G. W. HILL.

THE January number (volume 6, number 2) of the *Annals of Mathematics* contains the following papers: "The solutions of differential equations of the first order as functions of their initial values," by G. A. BLISS; "On the conformal representation of certain isosceles triangles upon the upper half-plane," by L. W. DOWLING; "Remarks on a proof that a continuous function is uniformly continuous," by N. J. LENNES.

THE publications *School Science* and *School Mathematics* (the latter originally a supplement of the former) have been combined and will be issued under new editorial management with the title *School Science and Mathematics*. Professor G. W. MYERS is departmental editor for mathematics. The associate editors include Professor C. E. COMSTOCK, Professor E. W. DAVIS, Mr. J. E. GOULD, Professor H. B. NEWSON, and Dr. ARTHUR SCHULTZE.

AT the meeting of the London mathematical society held on December 8, 1904, the following papers were read: by Major P. A. MACMAHON, "On a deficient multinomial expansion"; by Rev. F. H. JACKSON, "The application of basic numbers to Bessel's and Legendre's functions"; by Professor W. BURNSIDE, "On groups of order $p^\alpha q^\beta$"; by Dr. E. W. HOBSON, "On the failure of convergence of Fourier's series"; by Lt. Col. E. CUNNINGHAM, "An extension of Borel's exponential method of summation of divergent series applied to linear differential equations"; by Professor A. C. DIXON, "On the linear differential equation of the second order."

A REGULAR session of the Württembergischer mathematisch-naturwissenschaftlicher Verein was held at Stuttgart, December 4, 1904. The following papers were read: "Limiting values and limiting functions," by C. REUSCHLE (Stuttgart); "On experiments with the trifilar gravimeter," by A. SCHMIDT (Stuttgart); "On optical properties of the paraboloid," by V. KOMMERELL (Nürtingen). An animated discussion followed the reading of the first paper.

AT the Philadelphia meeting of the American association for the advancement of science, Professor C. M. WOODWARD was elected president of the association, Professor W. S. EICHELBERGER vice-president of section A, mathematics and astronomy, and Professor C. A. WALDO general secretary. The next meeting will be held at New Orleans in December, 1905.

A MATHEMATICAL section of the California teachers' association was organized December 26, 1904, at San Jose, California. Professor G. A. MILLER was elected president and Mr. J. F. SMITH secretary. The main object of the association is to arouse more interest in mathematical pedagogy by means of separate meetings for the discussion of recent mathematical movements.

AT a meeting of the State teachers' association held at Columbia, Mo. on December 27–29, 1904, a Missouri society of mathematics teachers was organized. The association will hold its meetings annually, beginning at Columbia on May 6.

A regular meeting of the Association of Ohio teachers of mathematics and science was held at Ohio State University, Columbus, December 29 and 30. One joint session and two sectional meetings were held each day. The following mathematical questions were discussed, the discussion being opened by two formal papers, one by a college representative, the other by a representative of the secondary schools: "What training in mathematics is desirable from the standpoint of the teacher of physics?", by Miss C. A. DAVIES and E. O. WEAVER; "Should the college entrance requirements in mathematics recommended by the AMERICAN MATHEMATICAL SOCIETY be adopted in Ohio?", by R. S. POND and H. HANCOCK; "What may be justly assumed as to preparation of high school graduates in algebra, both as to quantity and quality?", by J. D. HARLOR and E. M. MILLS; "What is a good outline for a semester's work in required algebra in the freshman year?", by F. ANDEREGG; "To what extent is a close correlation of the different branches of college mathematics desirable from the teacher's standpoint?", by W. H. WILSON; "Experimental methods in mathematics," by E. P. THOMPSON; "Is it desirable to introduce the modern treatment of synthetic geometry in the high school and college?", by M. E. GRABER; "What should be covered in one year's elective mathematics after the freshman mathematics?", by S. S. KELLAR; "Disposition of pupils deficient in mathematics," by T. L. FEENEY. The publication *School Science and Mathematics* was adopted as official organ. The members of the association were entertained at luncheon on Friday by the president and faculty of the university.

AT the annual meeting of the Paris academy of sciences, held on December 22, 1904, the following prizes were awarded for mathematical memoirs: Bordin prize (fr. 2000) to M. SERVANT for his memoir on the determination of surfaces applicable to the paraboloid of revolution which pass through a given contour; the Vaillant prize was divided between É. BOREL (fr. 3000) and L. BRICARD (fr. 1000); the Francœur prize (fr. 1000) to É. LEMOINE: the Poncelet prize (fr. 2000) to D. ANDRÉ; the Montyon prize in mechanics (fr. 700) to G.

RICHARD. The subjects for which these prizes were offered were announced in the BULLETIN, volume 10, page 323.

THE Nobel prize in physics was awarded to Lord Rayleigh, who has expressed his intention of giving it to Cambridge University, to promote research in physics.

OXFORD UNIVERSITY. — The following courses in mathematics are announced for the Hilary term, 1905: By Professor W. ESSON: Comparison of analytic and synthetic methods in the geometry of conics, two hours; Synthetic geometry of cubics, two hours. — By Professor E. B. ELLIOTT: Elements of elliptic functions, two hours; Substitutions and resolvents, one hour. — By Professor A. E. H. LOVE: Theory of potential, two hours; Elements of the calculus, two hours. — By Mr. J. W. RUSSELL: Algebra of quantics, two hours. — By Mr. P. J. KIRKBY: Higher algebra, one hour. — By Mr. A. L. DIXON: Calculus of finite differences, one hour. — By Mr. J. E. CAMPBELL: Geometry of surfaces, one hour. — By Mr. C. H. SAMPSON: Higher solid geometry, II, one hour. — By Mr. C. H. THOMPSON: Dynamics of a particle, three hours. — By Mr. H. T. GERRANS: Hydrodynamics, two hours. — By Mr. C. E. HASELFOOT: Theory of equations, one hour. — By Mr. C. LEUDESDORF: Geometry of inversions two hours.

PROFESSOR EXNER, of the University of Innsbruck, has retired from active work.

PROFESSOR RUSSJAN, of the University of Cracow, has been appointed professor of mechanics at the technical school of Lemberg.

DR. H. LIEBMANN has been promoted to an associate professorship of mathematics at the University of Leipzig.

DR. K. BOEHM has been promoted to an associate professorship of mathematics at the University of Heidelberg.

PROFESSOR P. STÄCKEL, of the University of Kiel, has been appointed professor of mathematics at the technical school at Hanover.

DR. P. FURTWÄNGLER has been appointed professor of mathematics at the agricultural academy at Poppelsdorf.

SIR NORMAN LOCKYER has been elected a corresponding member of the St. Petersburg academy of sciences.

IT is proposed to establish at the University of Liverpool a

memorial to the late Mr. R. W. H. T. HUDSON; it will probably take the form of an annual prize in geometry. Contributions to the fund may be sent to Mr. Alexander Muir, the University, Liverpool.

THE vacant position at the head of the department of mathematics in the College of the City of New York has been filled by the promotion of assistant professor J. R. SIM. Dr. P. L. SAUREL has been promoted to an assistant professorship of mathematics.

PROFESSOR JAMES W. MASON, who retired in 1903 after nearly twenty-five years service as professor of mathematics in the College of the City of New York, died at Easton, Pa., on January 10, at the age of sixty-nine years. Professor Mason was one of the earliest members of the AMERICAN MATHEMATICAL SOCIETY, having been admitted in May, 1890.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ARBICONE (A.). Intorno ad un problema di Leibniz. Memoria. Genova, Istituto Sordomuti, 1904. 8vo. 15 pp.

BAIRE (R.). Leçons sur les fonctions discontinues, professées au Collège de France. Rédigées par A. Denjoy. Paris, Gauthier-Villars, 1905. 8vo. 8 + 128 pp. Fr. 3.50

BÔCHER (M.). The fundamental conceptions and methods of mathematics. Address delivered before the Department of Mathematics of the International Congress of Arts and Sciences, St. Louis, Sept. 20, 1904. (*Bulletin of the American Mathematical Society*, Vol. 11, pp. 115–135.)

BRÖHL (J.). Doppelt unendliche Kurvenscharen, gebildet von Schraubenlinien auf den Flächen der Rotationsflächenschar $x^2 + y^2 = u^2c^2z$. (Diss.) Leipzig, 1904. 8vo. 53 pp. M. 2.00

BÜRKLEN (O. T.). Formelsammlung und Repetitorium der Mathematik, enthaltend die wichtigsten Formeln und Lehrsätze der Arithmetik, Algebra, algebraischen Analysis, ebenen Geometrie, Stereometrie, ebenen und sphärischen Trigonometrie, mathematischen Geographie, analytischen Geometrie der Ebene und des Raumes, der Differential- und Integralrechnung. 3te, durchgesehene Auflage. Leipzig, Göschen, 1904. 16mo. 227 pp. Cloth. (Sammlung Göschen, No. 51.) M. 0.80

CIRILLO (C.). Sopra una trasformazione di second'ordine delle funzioni ellittiche. Nota. Chieti, Sciullo, 1904. 16mo. 18 pp.

COMPOSTO (S.). Sulla trasformazione dei radicali sovrapposti. Bologna, Cuppini, 1904. 8vo. 55 pp. Fr. 1.50

CUNNINGHAM (A.). Quadratic partitions. London, Hodgson, 1904. 8vo. Cloth. 12s.

DENJOY (A.). See BAIRE (R.).

FUBINI (G.). Sui gruppi di proiettività. Nota. Catania, Galàtola, 1904. 4to. 8 pp.

GIBSON (G.). An introduction to the calculus, based on graphical methods. New York, Macmillan, 1904. 8vo. 13 + 225 pp. Cloth. $0.90

JOHNSON (W. W.). Elementary treatise on the differential calculus, founded on the method of rates. New York, Wiley, 1904. 8vo. 14 + 406 pp. Cloth. $3.00

LAMBERT (P. A.). Answers to problems in the analytic geometry. New York, Macmillan, 1904. 16mo. 10 pp. $0.30

——. Answers to problems in the differential and integral calculus. New York, Macmillan, 1904. 16mo. $0.30

NAETSCH (E.). See SCHLÖMILCH (O.).

PALMIERI (F. S.). Studio sui fondamenti di una metrica subordinata alla proiettività nelle forme di prima specie, seguito da considerazioni su alcune questioni geometriche fondamentali. Vicenza, Fabris, 1904. 8vo. 114 pp. Fr. 3.00

ROSSO (U.). Sulla generalizzazione dei metodi di rappresentazione in geometria descrittiva. Nota. Genova, Ciminago, 1904. 8vo. 23 pp.

SCHLÖMILCH (O.). Uebungsbuch zum Studium der höheren Analysis. Teil I: Aufgaben aus der Differentialrechnung. 5te Auflage. Bearbeitet von E. Naetsch. Leipzig, Teubner, 1904. 8vo. 8 + 372 pp. Cloth. M. 8.00

SHAPOSHNIKOV. Elements of higher analysis. 2d edition. Moscow. 1904. 8vo. 141 pp. (Russian.) M. 3.00

VRIES (H. DE). Die Lehre von der Zentralprojektion im vierdimensionalen Raume. Leipzig, Göschen, 1905. 8vo. 78 pp. M. 3.00

II. ELEMENTARY MATHEMATICS.

BÖKELER (A.). Stereometrische Aufgaben aus den Reifeprüfungen der Gymnasialabiturienten. Ravensburg, Alber, 1904. 8vo. 3 + 88 pp. M. 1.20

BOSS (A.). Elementary algebra, translated by N. de George. St. Petersburg, 1904. 8vo. 519 pp. (Russian.) M. 7.50

BRYAN (G. H.) and DEAKIN (R.). Certificate algebra. London, Clive, 1904. 8vo. 300 pp. Cloth. 2s. 6d.

BÜTZBERGER (F.). Lehrbuch der ebenen Trigonometrie, mit vielen Aufgaben und Anwendungen, für Gymnasien und technische Mittelschulen. Resultate und Auflösungen. Zürich, Füssli, [1904]. 8vo. 24 pp. M. 1.50

CARL (L.). Algebraische Aufgaben zur Einführung in die Arithmetik. 2te Auflage. Oldenburg, Stalling, 1905. 8vo. 4 + 48 pp. M. 0.60

CHAILAN (E.). Algèbre, à l'usage des élèves de l'enseignement secondaire (premier et deuxième cycles, sections A et B). Paris, Poussielgue, 1904. 16mo. 2 + 74 pp.

DEAKIN (R.). Algebra. King's Scholarship edition. With papers set at Scholarship examination for four years. London, Clive, 1904. 8vo. 336 pp. Cloth. 2s. 6d.

——. See BRYAN (G. H.).

DURELL (F.). Plane geometry. Harrisburg, Pa., Myers, Fishel and Co., 1904. 16mo. 320 pp. Half-leather. $0.75

FONT (N.) et HUGON (C.). Algèbre, à l'usage des élèves du premier cycle, des élèves du deuxième cycle (sections A et B, programmes du 31 mai 1902), et des élèves de l'enseignement primaire supérieur. Marseilles, Ferran, 1904. 16mo. 8 + 290 pp.

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HALL (H. S.) and STEVENS (F. H.). School geometry. Part I–VI. Containing plane and solid geometry, treated theoretically and graphically. London, Macmillan, 1904. 8vo. 468 pp. Cloth. 4s. 6d.

——. School geometry. Parts IV, V, VI. London, Macmillan, 1904. 8vo. 243 pp. Cloth. 2s. 6d.

HUGON (C.). See FONT (N.).

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LACKEMANN (C.). Die Elemente der Geometrie. Ein Lehr- und Uebungsbuch für den geometrischen Unterricht an sechsklassigen höheren Lehranstalten. 7te, verbesserte Auflage, unter Berücksichtigung der preussischen Lehrpläne vom Jahre 1901 bearbeitet von Kreuschmer. Teil 1: Planimetrie. Breslau, Hirt, 1904. 8vo. 128 pp. Boards. M. 1.30

MARKS (C. J.). Mathematical questions and solutions from "Educational Times." London, Hodgson, 1904. New issue. Vol. 6. 8vo. 6s. 6d.

MARTINAUD (A.). Eléments d'arithmétique et de géométrie, à l'usage de l'enseignement secondaire, rédigés conformément aux programmes officiels de 1902. 3e édition. Paris, Belin, 1905. 12mo. 235 pp.

NARDI (P.). Geometria pratica ad uso degli alunni delle scuole tecniche e professionali. Parte II: Stereometria. Livorno, Giusti, 1904. 16mo.

OTT (K. VON). Tafeln der Logarithmen und anderer beim mathematischen Unterrichte unentbehrlichen Zahlenwerte für Mittelschulen. 5te Auflage. Prag, Calve, 1905. 8vo. 22 + 160 pp. Boards. M. 1.80

QUINN (J. J.). A Socratic study of plane geometry. Syracuse, Bardeen, 1904. 12mo. 272 pp. Buckram. $1.00

ROSENBERG (K.). Methodisch geordnete Sammlung von Aufgaben aus der Planimetrie und Stereometrie für Lehrer- und Lehrerinnen-Bildungsanstalten, sowie für andere gleichgestellte Lehranstalten. 3te Auflage. Wien, Hölder, 1904. 8vo. 4 + 168 pp. Cloth. M. 1.70

STEVENS (F. H.). See HALL (H. S.).

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TESTI (G. M.). Corso di matematiche ad uso delle scuole secondarie superiori e più specialmente degli istituti tecnici. Vol. VII (ultimo). Livorno, Giusti, 1904. 8vo.

III. APPLIED MATHEMATICS.

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CASTLE. Workshop mathematics. Parts I and II. Complete in one volume. New York, Macmillan, 1904. 12mo. 8 + 154 + 6 + 175 pp. Cloth. $0.65

FERRIS (C. E.). Elements of descriptive geometry. Knoxville, Tenn., Gaut-Ogden Co., 1904. 12mo. 7 + 127 pp. Cloth. $1.50

GALILEI (G.). Le opere. Edizione nazionale sotto gli auspici di Sua Maestà il Re d'Italia. Vol. XV: Carteggio, 1633; Indice cronologico delle lettere contenute nel vol. XV; Indice alfabetico delle lettere contenute nel vol. XV. Firenze, Barbèra, 1904. 4to.

HARDY (E.). The elementary principles of graphic statics; specially prepared for students of science and technical schools and those entering for the examinations of the Board of Education in building construction, machine construction, drawing, applied mechanics, and for other similar examinations. New York, Van Nostrand, 1904. 12mo. 3 + 165 pp. Cloth. $1.50

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RIGHI (A.). Modern theory of physical phenomena; radio-activity. ions, electrons; authorized translation by A. Trowbridge. New York. Macmillan, 1904. 12mo. 13 + 165 pp. Cloth. $1.10

RODET (J.). Résistance, inductance et capacité. Paris, Gauthier-Villars, 1904. 8vo. 10 + 258 pp. Fr. 7.00

RUSSELL (A.). Treatise on the theory of alternating currents. Vol. I. London, Clay, 1904. 420 pp. Cloth. 12s.

SCHWAMB (P.) and MERRILL (A. L.). Elements of mechanism. New York, Wiley, 1904. 8vo. 6 + 264 pp. Cloth. $3.00

SOTTA (F. M.). La meccanica dell' infinito. Teoria nuova. Milano, 1904. 8vo. 24 + 215 pp. M. 12.50

STODDARD (C. W.). Geometric problems; their application to constructive drawings. New York, Prang Educational Co., 1904. 12mo. 79 pp. $0.30

STODOLA (A.). The steam turbine; translated by L. C. Loewenstein. New York, Van Nostrand, 1904. 8vo. Cloth. $4.50

TETMAJER (L. VON). Die angewandte Elastizitäts- und Festigkeitslehre. Auf Grundlage der Erfahrung bearbeitet. 3te, umgearbeitete Auflage. Wien, Deuticke, 1905. 8vo. 16 + 618 pp., 11 plates. M. 16.00

TROWBRIDGE (A.). See RIGHI (A.).

VONDERLINN (J.). Schattenkonstruktionen. Leipzig, Göschen, 1904. 16mo. 118 pp. Cloth. (Sammlung Göschen, No. 236.) M. 0.80

WHITTAKER (E. T.). Treatise on the analytical dynamics of particles and rigid bodies. Introduction to the problem of three bodies. London, Clay, 1904. 8vo. 428 pp. Cloth. 12s. 6d.